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## Intensity Scintillations of an EM Wave in Extremely Strong Turbulence

RONALD L. FANTE

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*Walter Rotman*

APPROVED:

Walter Rotman  
Project Engineer

APPROVED:

*Carlyle J. Sletten*  
CARLYLE J. SLETTEN, Chief  
Electromagnetic Sciences Division

FOR THE COMMANDER:

*John P. Hues*

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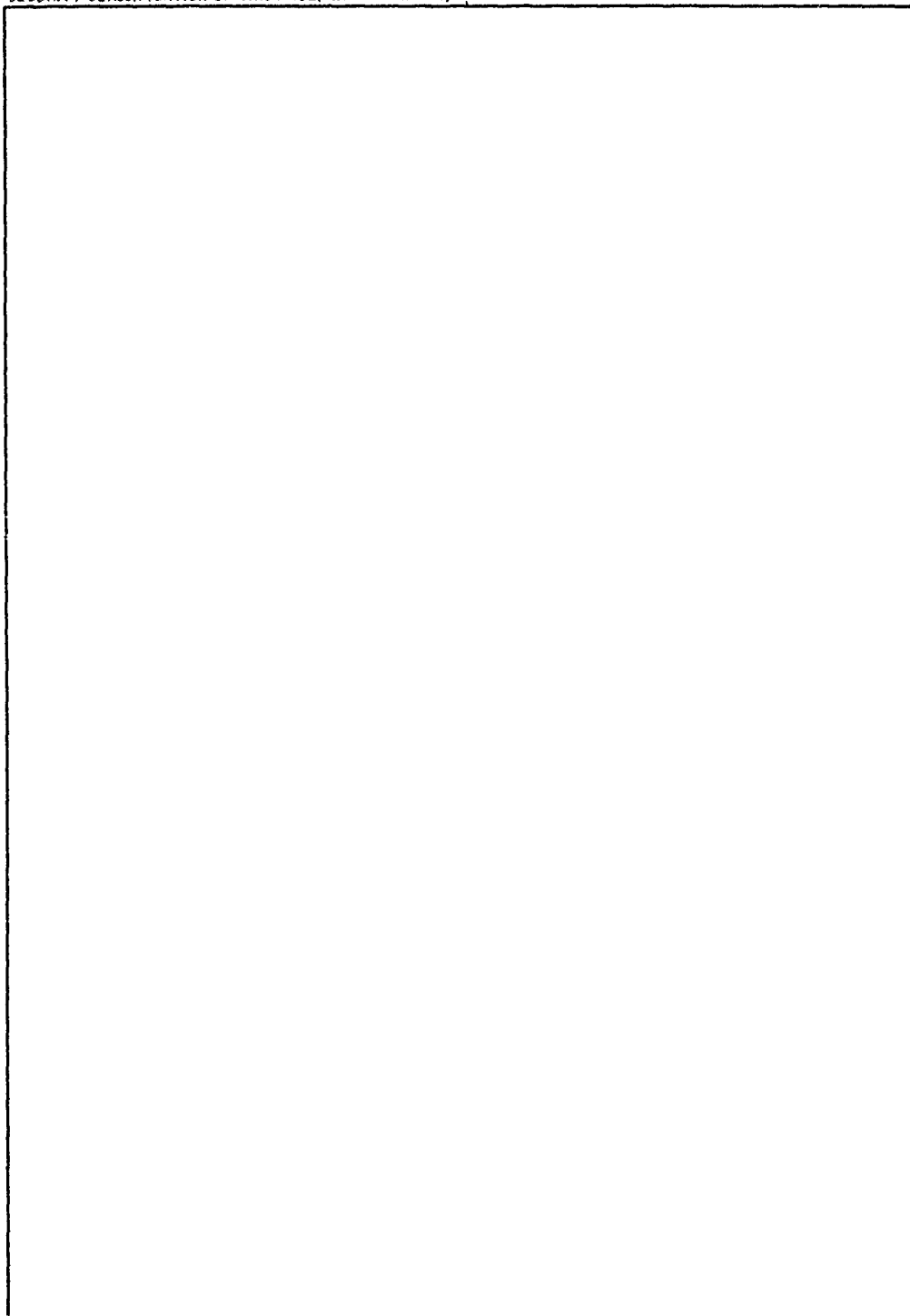
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## Intensity Scintillations of an EM Wave in Extremely Strong Turbulence

### 1. INTRODUCTION

Recent theoretical research<sup>1-4</sup> on the propagation of electromagnetic waves in extended strong turbulence indicates that the electric field obeys gaussian statistics in the limit when the strength of the turbulence becomes infinite. In fact, it has been shown<sup>5</sup> that theoretical solutions, which have as their basis the assumption of gaussian statistics for extremely strong turbulence, give excellent agreement with measured data for the intensity fluctuations in the case of moderately strong turbulence. Therefore, even though measured data indicate that the field statistics are nearly log-normal in moderately strong turbulence, there is good reason to believe that the field statistics are gaussian when the turbulence is

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1. de Wolf, D. (1974) Waves in turbulent air: A phenomenological model, Proc. IEEE, 62:1523-1529.
2. Shishov, V. (1972) Strong fluctuations of the intensity of a plane wave propagating in a random refractive medium, Soviet Phys. JETP 34:744-748.
3. Prokhorov, A., Bunkin, F., Gochelashvily, K., and Shishov, V. (1975) Laser irradiance propagation in turbulent media, Proc. IEEE 63:790-811.
4. Fante, R. (1975) Some new results on the propagation of electromagnetic waves in strongly turbulent media, IEEE Trans. AP-23:382-385.
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extremely strong. Unfortunately, there are, at present, no measurements of the field statistics in extremely strong turbulence.

In this paper, we shall assume that the field statistics of a point source approach gaussian in extremely strong turbulence. By using that assumption, we shall then calculate expressions for the averaged intensity, intensity correlation and scintillation index for a source of arbitrary shape; the source may be either coherent or incoherent. We find that the scintillation index always approaches unity. This result appears to disagree with some recent calculations by Rumsey<sup>6</sup> who found that the scintillation index for an incoherent source approaches zero in extremely strong turbulence. The reason for this apparent discrepancy is investigated.

## 2. COHERENT SOURCE

Let us first consider the scintillations in the received signal from a coherent source, such as a laser. By using the extended Huygens-Fresnel principle, we can show that the field distribution  $e(\underline{\rho})$  in the plane  $x$  is related to the source distribution  $e_o(\underline{\rho}_1)$  via<sup>7</sup>

$$e(\underline{\rho}) = \frac{k}{2\pi ix} \iint_{-\infty}^{\infty} d^2\rho_1 e_o(\underline{\rho}_1) \exp \left[ i \frac{k}{2x} (\underline{\rho} - \underline{\rho}_1)^2 + \psi(\underline{\rho}, \underline{\rho}_1) \right] \quad (1)$$

where  $k$  is the signal wavenumber and  $\psi(\underline{\rho}, \underline{\rho}_1)$  is the additional complex phase due to turbulence of a spherical wave propagating from  $(0, \underline{\rho}_1)$  to  $(x, \underline{\rho})$ . We now desire to obtain the long-term averaged intensity  $\langle I(\underline{\rho}) \rangle$  and the long-term averaged intensity correlation  $\langle I(\underline{\rho}) I(\underline{\rho}') \rangle$ . By long-term averaged, we mean that the average is taken over times long compared with the coherence time of the turbulence. For example, for a beam of diameter  $D$  we take the average over times long compared with  $D/V$ , where  $V$  is the transverse velocity of the turbulent eddies. By using Eq. (1) we find it straightforward to show that

6. Rumsey, V. (1975) Scintillations due to an extended source in extended strong turbulence, Digest of the 1975 Annual USNC/URSI Meeting, Nat'l Academy of Sciences, Wash., D.C. 20418, p 246.
7. Feizulin, Z., and Kravtsov, Y. (1967) Broadening of a laser beam in a turbulent medium, Radiophys. Quantum Electron 10:33-35.



$$\begin{aligned}
\langle I(\underline{\rho}) \rangle &\equiv \langle e(\underline{\rho}) e^*(\underline{\rho}) \rangle \\
&= \left( \frac{k}{2\pi x} \right)^2 \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) \\
&\quad \cdot \exp \left\{ i \frac{k}{x} \underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) + \frac{ik}{2x} (\rho_1^2 - \rho_2^2) \right\} \\
&\quad \cdot \langle \exp[ \psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_2) ] \rangle
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\langle I(\underline{\rho}) I(\underline{\rho}') \rangle &= \left( \frac{k}{2\pi x} \right)^4 \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 d^2 \rho_4 e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) e_o(\underline{\rho}_3) e_o^*(\underline{\rho}_4) \\
&\quad \cdot \exp \left\{ i \frac{k}{x} \underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) + i \frac{k}{x} \underline{\rho}' \cdot (\underline{\rho}_3 - \underline{\rho}_4) + \frac{ik}{2x} (\rho_1^2 - \rho_2^2 + \rho_3^2 - \rho_4^2) \right\} \\
&\quad \cdot \langle \exp[ \psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_2) + \psi(\underline{\rho}', \underline{\rho}_3) + \psi^*(\underline{\rho}', \underline{\rho}_4) ] \rangle
\end{aligned} \tag{3}$$

Let us now employ the ergodic hypothesis so that the time average is replaced by an ensemble average. In this case it can be shown that<sup>7</sup>

$$\langle \exp[ \psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}', \underline{\rho}_2) ] \rangle = \exp \left\{ -\frac{1}{2} D_1(\underline{\rho} - \underline{\rho}', \underline{\rho}_1 - \underline{\rho}_2) \right\} \tag{4}$$

where for a Kolmogorov spectrum for the index of refraction fluctuations, we have<sup>8</sup>

$$D_1(\underline{\xi}, \underline{\eta}) = k^2 \int_0^x dx' C_n^2(x') g(\beta) \tag{5}$$

8. Kon, A., and Feizulin, Z. (1970) Fluctuations in the parameters of spherical waves propagating in a turbulent atmosphere, Radiophys. Quantum Electron. 13:51-53.

with

$$\beta \equiv \left| \xi \frac{x'}{x} + \eta \left( 1 - \frac{x'}{x} \right) \right|, \quad (6)$$

$$g(\beta) \simeq 1.8 \beta^2 K_m^{1/3} \quad \text{if} \quad \beta \ll 1/K_m, \quad (7a)$$

$$g(\beta) \simeq 2.92 \beta^{5/3} \quad \text{if} \quad \beta \gg 1/K_m, \quad (7b)$$

and  $C_n^2$  is the index of refraction structure constant and  $K_m = 5.91/l_0$ , where  $l_0$  is the inner scale size of the turbulence.

We now assume that the turbulence is sufficiently strong so that the function  $v(\underline{\rho}, \underline{\rho}_1) \equiv \exp \psi(\underline{\rho}, \underline{\rho}_1)$  is a gaussian random variable. As discussed earlier, the general theoretical solutions of Prokhorov et al.<sup>3</sup> and Fante,<sup>4</sup> as well as the physical model of de Wolf,<sup>1</sup> suggest that this should be the case for sufficiently large values of  $k^{7/6} C_n^2 x^{11/6}$ . For gaussian point-source statistics we have

$$\begin{aligned} & \langle \exp[\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_2) + \psi(\underline{\rho}', \underline{\rho}_3) + \psi^*(\underline{\rho}', \underline{\rho}_4)] \rangle \\ &= \langle \exp[\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_2)] \rangle \langle \exp[\psi(\underline{\rho}', \underline{\rho}_3) + \psi^*(\underline{\rho}', \underline{\rho}_4)] \rangle \\ &+ \langle \exp[\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}', \underline{\rho}_4)] \rangle \langle \exp[\psi(\underline{\rho}', \underline{\rho}_3) + \psi^*(\underline{\rho}, \underline{\rho}_2)] \rangle \\ &= \exp \left[ -\frac{1}{2} D_1(0, \underline{\rho}_1 - \underline{\rho}_2) - \frac{1}{2} D_1(0, \underline{\rho}_3 - \underline{\rho}_4) \right] \\ &+ \exp \left[ -\frac{1}{2} D_1(\underline{\rho} - \underline{\rho}', \underline{\rho}_1 - \underline{\rho}_4) - \frac{1}{2} D_1(\underline{\rho}' - \underline{\rho}, \underline{\rho}_3 - \underline{\rho}_2) \right]. \end{aligned} \quad (8)$$

If we now use Eqs. (4) and (8) in Eqs. (2) and (3) we get

$$\begin{aligned} \langle I(\underline{\rho}) \rangle &= \left( \frac{k}{2\pi x} \right)^2 \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) \exp \left\{ -\frac{1}{2} D_1(0, \rho_1 - \rho_2) \right\} \\ &\cdot \exp \left\{ i \frac{k}{x} \rho \cdot (\underline{\rho}_1 - \underline{\rho}_2) + i \frac{k}{2x} (\rho_1^2 - \rho_2^2) \right\} \end{aligned} \quad (9)$$

and

$$\begin{aligned}
\langle I(\underline{\rho}) I(\underline{\rho}') \rangle &= \left( \frac{k}{2\pi x} \right)^4 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 d^2 \rho_4 e_{o1}(\underline{\rho}_1) e_{o2}^*(\underline{\rho}_2) e_{o3}(\underline{\rho}_3) e_{o4}^*(\underline{\rho}_4) \\
&\cdot \exp \left\{ i \frac{k}{x} \underline{\rho} \cdot (\underline{\rho}_1 - \underline{\rho}_2) + i \frac{k}{x} \underline{\rho}' \cdot (\underline{\rho}_3 - \underline{\rho}_4) + \frac{ik}{2x} (\rho_1^2 - \rho_2^2 + \rho_3^2 - \rho_4^2) \right. \\
&\cdot \left\{ \exp \left[ -\frac{1}{2} D_1(0, \underline{\rho}_1 - \underline{\rho}_2) - \frac{1}{2} D_1(0, \underline{\rho}_3 - \underline{\rho}_4) \right] \right. \\
&\left. \left. + \exp \left[ -\frac{1}{2} D_1(\underline{\rho} - \underline{\rho}', \underline{\rho}_1 - \underline{\rho}_4) - \frac{1}{2} D_1(\underline{\rho}' - \underline{\rho}, \underline{\rho}_3 - \underline{\rho}_2) \right] \right\} \right\} \quad (10)
\end{aligned}$$

Now let us consider the scintillation index

$$m^2 = \frac{\langle I^2(\underline{\rho}) \rangle - \langle I(\underline{\rho}) \rangle^2}{\langle I(\underline{\rho}) \rangle^2} \quad (11)$$

Upon setting  $\underline{\rho}' = \underline{\rho}$  and substituting Eqs. (10) and (9) into Eq. (11), we find that

$$m^2 = 1 \quad (12)$$

Therefore, for an arbitrary coherent source  $m^2 \rightarrow 1$  as the strength of turbulence approaches infinity. This result is a generalization of earlier results which showed<sup>4</sup> that  $m^2 \rightarrow 1$  for a plane wave.

### 3. INCOHERENT SOURCE

We now consider the case when the source is incoherent. We shall assume that the coherence time  $\tau_s$  of the source is much smaller than the coherence time  $\tau_a$  of the atmospheric turbulence. Therefore, we shall first obtain the average of  $I(\underline{\rho})$  and  $I(\underline{\rho}) \cdot I(\underline{\rho}')$  over times which are long compared with  $\tau_s$  but short compared with  $\tau_a$ . For an incoherent source containing very many coherence areas (the coherence area is of linear dimension  $c\tau_s$  where  $c$  is the speed of light), it can be shown that<sup>9</sup>

9. Beran, M., and Parrent, G. (1964) Theory of Partial Coherence, Englewood Cliffs, N.J., Prentice Hall, Englewood Cliffs, N.J.

$$\overline{e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2)} \equiv \frac{1}{T} \int_0^T e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) dt \simeq \hat{I}_s(\underline{\rho}_1) \delta(\underline{\rho}_1 - \underline{\rho}_2) \quad (13)$$

where

$$\tau_s \ll T \ll \tau_a.$$

Also

$$\begin{aligned} \overline{e_o(\underline{\rho}_1) e_o^*(\underline{\rho}_2) e_o(\underline{\rho}_3) e_o^*(\underline{\rho}_4)} &= \hat{I}_s(\underline{\rho}_1) \hat{I}_s(\underline{\rho}_3) \delta(\underline{\rho}_1 - \underline{\rho}_2) \delta(\underline{\rho}_3 - \underline{\rho}_4) \\ &+ \hat{I}_s(\underline{\rho}_1) \hat{I}_s(\underline{\rho}_3) \delta(\underline{\rho}_1 - \underline{\rho}_4) \delta(\underline{\rho}_2 - \underline{\rho}_3) \end{aligned} \quad (14)$$

where  $I_s$  is the intensity distribution of the source. Upon using Eqs. (13) and (14) in Eqs. (2) and (3), one finds it easy to show that the average over times  $T$  such that  $\tau_s \ll T \ll \tau_a$  is

$$\overline{I(\underline{\rho})} = \left( \frac{k}{2\pi x} \right)^2 \iint_{-\infty}^{\infty} d^2 \rho_1 \hat{I}_s(\underline{\rho}_1) \exp [\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_1)] \quad (15)$$

and

$$\begin{aligned} \overline{I(\underline{\rho}) I(\underline{\rho}')}, &= \overline{I(\underline{\rho})} \overline{I(\underline{\rho}')} + \left| \left( \frac{k}{2\pi x} \right)^2 \iint_{-\infty}^{\infty} d^2 \rho_1 \hat{I}_s(\underline{\rho}_1) \right. \\ &\cdot \left. \exp \left\{ i \frac{k}{x} (\underline{\rho} - \underline{\rho}') \cdot \underline{\rho}_1 + \psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}', \underline{\rho}_1) \right\} \right|^2. \end{aligned} \quad (16)$$

Equation (16) is the generalized version of the Brown-Twiss formula and for no turbulence ( $\psi = 0$ ) it reduces to the usual Brown-Twiss result.

Next let us obtain the average over times which are long compared with  $\tau_a$ . If we again assume ergodicity, so that we replace the time average by an ensemble average, and then use Eqs. (4) and (8) to calculate  $\langle \exp[\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_1)] \rangle$  and  $\langle \exp[\psi(\underline{\rho}, \underline{\rho}_1) + \psi^*(\underline{\rho}', \underline{\rho}_1) + \psi^*(\underline{\rho}, \underline{\rho}_2) + \psi(\underline{\rho}', \underline{\rho}_2)] \rangle$  we obtain\* for the long-time averages of  $I(\underline{\rho})$  and  $I(\underline{\rho}) I(\underline{\rho}')$

\*In writing Eqs. (17) and (18) we have assumed that the measuring apparatus can essentially measure the instantaneous values of  $I$ ; that is, we assume that the apparatus response time is short compared with the source coherence time.

$$\langle I(\underline{\rho}) \rangle = \left( \frac{k}{2\pi x} \right)^2 \int_{-\infty}^{\infty} d^2 \rho_1 \hat{I}_s(\underline{\rho}_1) \quad (17)$$

$$\begin{aligned} \langle I(\underline{\rho}) I(\underline{\rho}') \rangle &= \left( \frac{k}{2\pi x} \right)^4 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 \hat{I}_s(\underline{\rho}_1) \hat{I}_s(\underline{\rho}_2) \left[ 1 + \exp \{ -D_1(\underline{\rho} - \underline{\rho}', \underline{\rho}_1 - \underline{\rho}_2) \} \right] \\ &\quad + \left( \frac{k}{2\pi x} \right)^4 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 \hat{I}_s(\underline{\rho}_1) \hat{I}_s(\underline{\rho}_2) \exp \left[ i \frac{k}{x} (\underline{\rho} - \underline{\rho}') \cdot (\underline{\rho}_1 - \underline{\rho}_2) \right] \\ &\quad \cdot \left[ \exp \{ -D_1(\underline{\rho} - \underline{\rho}', 0) \} + \exp \{ -D_1(0, \underline{\rho}_1 - \underline{\rho}_2) \} \right] . \end{aligned} \quad (18)$$

Equations (17) and (18) represent the averages over times longer than  $\tau_a$ , assuming that the field statistics for a point source become gaussian (rather than log-normal, as is the case in weak turbulence); in the limit of very strong turbulence it is appropriate to use the form in Eq. (7a) for  $D_1$  so that

$$D_1(0, \rho_1 - \rho_2) \simeq Q |\rho_1 - \rho_2|^2, \quad (19)$$

where

$$Q \equiv \frac{1.8 K_m^{1/3} k^2}{x^2} \int_0^x dx' x'^2 C_n^2(x').$$

If we use a source distribution of the form  $\hat{I}_s(\rho_1) = \exp(-\rho_1^2/2a^2)$  it is readily found from Eqs. (17) and (18) that (for  $Qa^2 \gg 1$ )

$$\langle I \rangle = 2\pi \left( \frac{ka}{2\pi x} \right)^2, \quad (20a)$$

$$\langle I^2 \rangle = 2! \langle I \rangle^2 \left\{ 1 + \frac{1}{1 + 4Qa^2} \right\}. \quad (20b)$$

By a more complex calculation we also find that

$$\langle I^3 \rangle = 3! \langle I \rangle^3 \left\{ 1 + \frac{3}{1 + 4Qa^2} + \frac{2}{(1 + 3Qa^2)^2} \right\}, \quad (20c)$$

and so on. From Eqs. (20a) to (20c) it is clear that the statistics of the beam are not necessarily gaussian even though we have assumed that the point source statistics are purely gaussian. This is clear because for pure gaussian field statistics we would have  $\langle I^2 \rangle = 2! \langle I \rangle^2$ ,  $\langle I^3 \rangle = 3! \langle I \rangle^3$ , and so on, whereas from Eqs. (20a) to (20c) we see that this is the case only in the limit of a plane wave ( $a \rightarrow \infty$ ). For the case of finite (but large)  $Q$ , the field statistics of a finite beam are almost, but not quite, gaussian.\*

We can now use Eqs. (20a) and (20b) to calculate the scintillation index  $m^2$ . We find that

$$m^2 = 1 + \frac{2}{1 + 4Qa^2} \quad (21)$$

from which we see that  $m^2 \rightarrow 1$  as  $Q \rightarrow \infty$ . Note that Eq. (21) is valid only for  $Qa^2 \gg 1$  because of our assumption of very strong turbulence. It is interesting to note that the results in Eqs. (13) to (21) can also, with suitable modifications, be used to describe the scatter of a beam from a diffuse surface located in a strongly turbulent medium.

Before concluding this section, we find it interesting to see what would be obtained if we could not measure the instantaneous value of  $I(\rho)$  but could measure only the value of  $I(\rho)$  averaged over a time greater than the coherence time  $\tau_s$  of the star (but short compared with  $\tau_a$ ). As before, let us denote this value by  $\bar{I}(\rho)$ . If we took the long-time average of  $\bar{I}(\rho) \bar{I}(\rho')$  (as opposed to the average of the product  $I(\rho) I(\rho')$  of the instantaneous intensities) we get from Eq. (15)

$$\langle \bar{I}(\rho) \bar{I}(\rho') \rangle = \left( \frac{k}{2\pi x} \right)^4 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2 \rho_1 d^2 \rho_2 \hat{I}_s(\rho_1) \hat{I}_s(\rho_2) [1 + \exp \{-D_1(\rho - \rho', \rho_1 - \rho_2)\}] \quad (22)$$

If the measurements are made in the manner described here, the equivalent scintillation index  $\bar{m}^2 = \langle \bar{I}^2 \rangle / \langle \bar{I} \rangle^2 - 1$  is, for the form of  $D_1$  assumed in Eq. (20)

$$\bar{m}^2 = \frac{1}{1 + 4Qa^2} \quad (23)$$

which approaches zero as  $Q \rightarrow \infty$ . A result equivalent to Eq. (23) has been obtained by Rumsey.<sup>6</sup>

\*That is, for gaussian field statistics the probability distribution of the intensity is  $p(I) = \frac{1}{\langle I \rangle} \exp \left( -\frac{I}{\langle I \rangle} \right)$  which leads to moments  $\langle I^N \rangle = N! \langle I \rangle^N$ .

Therefore, the result we obtain for the scintillation index of an incoherent source depends on how the intensity measurements are made. If the measurement apparatus can obtain essentially instantaneous values (that is, time response short compared with source coherence time  $\tau_s$ ) of  $I$  and these are used to obtain  $\langle I^2 \rangle$  and  $\langle I \rangle$  then the measured scintillation index approaches unity in extremely strong turbulence. However, if the measurement apparatus can measure only the value of  $I$  averaged over a time long compared with the coherence time of the source, we find that the measured scintillation index will approach zero.

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